Review of HW 5
NLP in GAMS

For ARE 521 QUANTITATIVE TECHNIQUES

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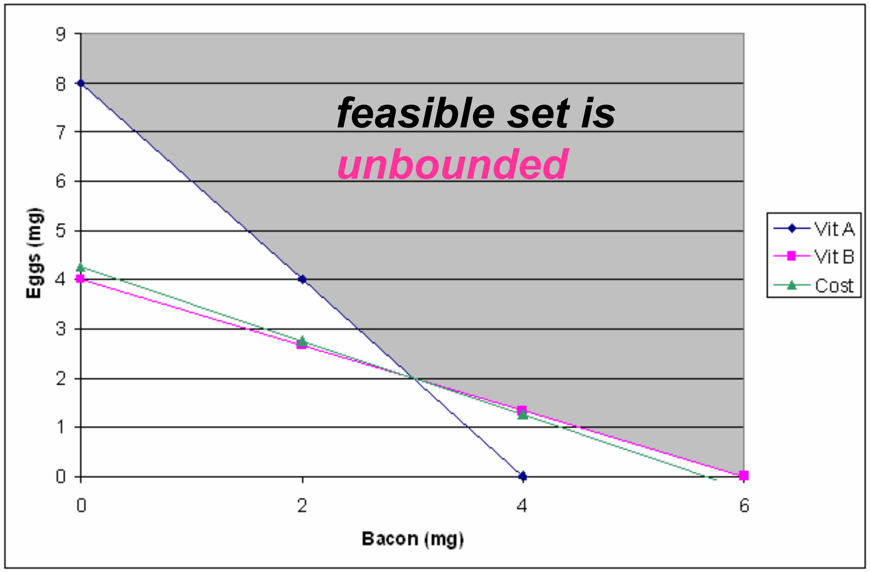
Sets

- Set - collection of distinct objects
  - Students in class
  - Feasible solutions

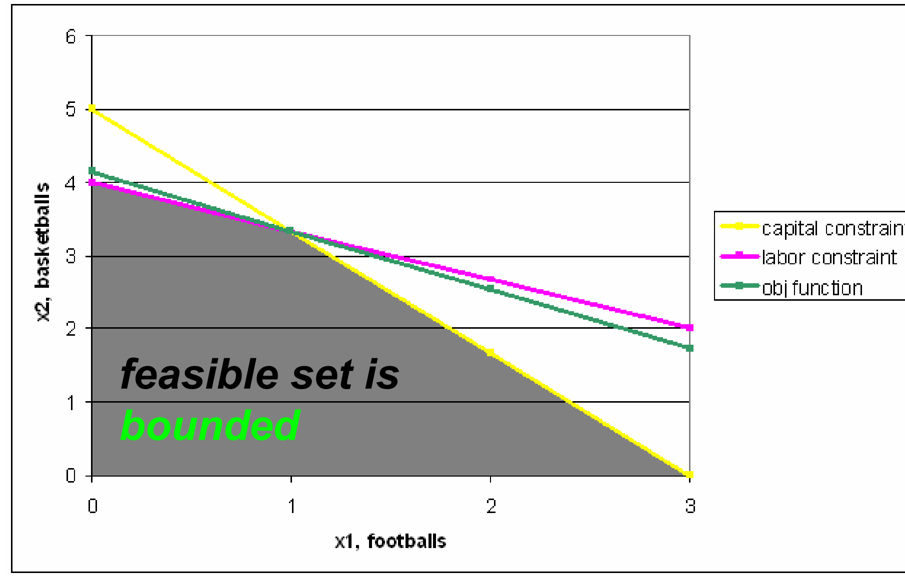
- Objects are *members* of the set

Bounded and Unbounded Sets

Minimize breakfast cost subject to vitamin requirements

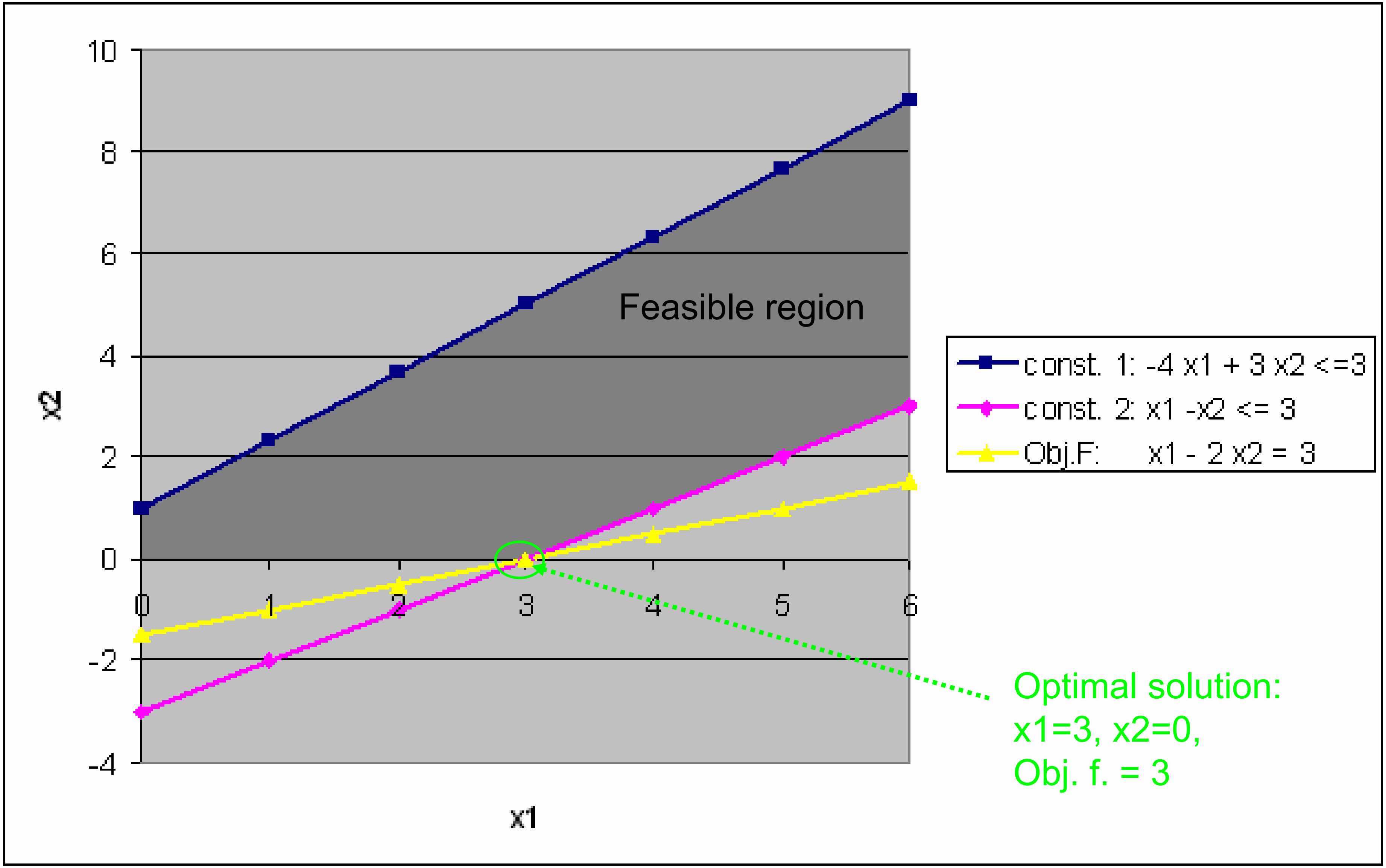


Maximize GM subject to resource availability

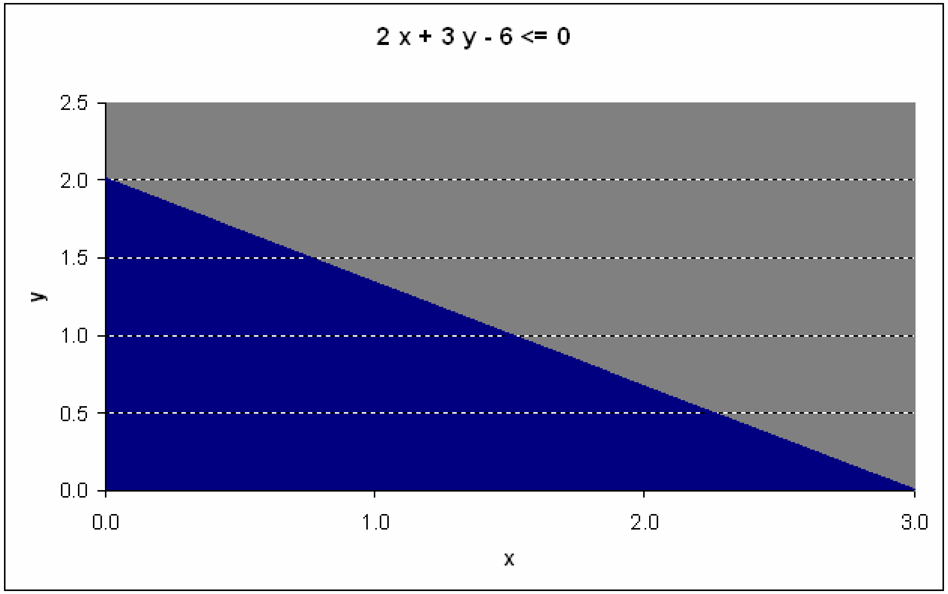


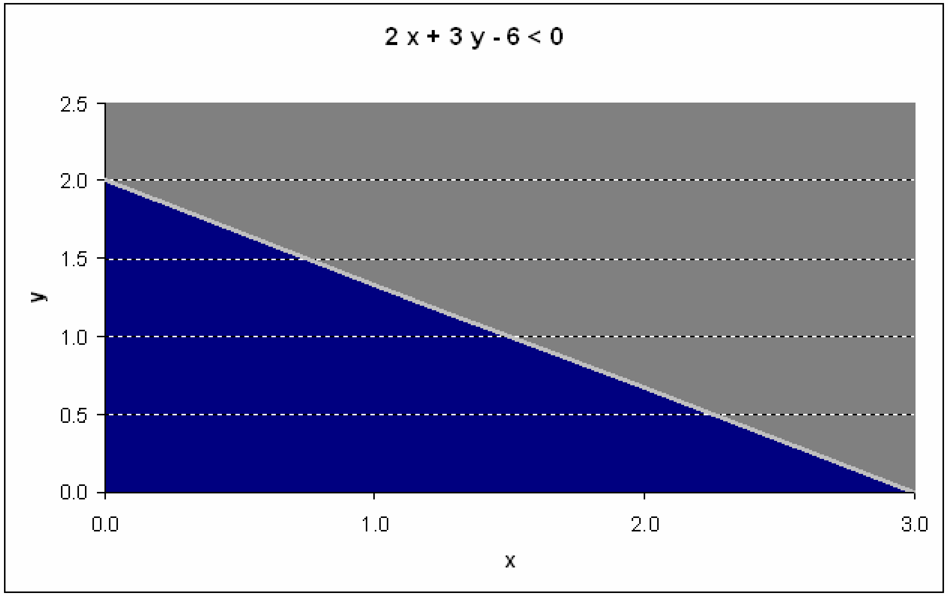
- Set is ***bounded*** - A set that can be placed inside a sufficiently large circle or sphere.

Assignment 5, problem 3



Open and Closed Sets



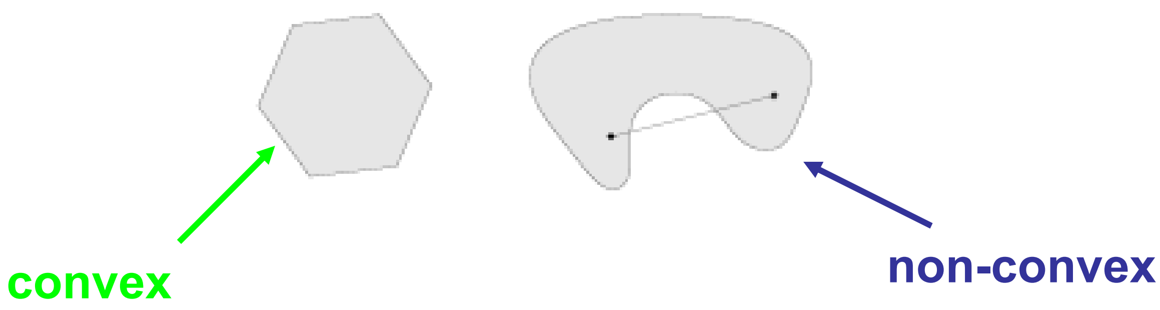


- Set *D* is *open* if for each point *x* in *D* there is an open neighborhood in D centered at x.

- The set $\{(x,y\}: 2x + 3y - 6 < 0\}$ is open and $\{(x,y\}: 2x + 3y - 6 \leq 0\}$ is closed.

Convex and Non-Convex Sets

- A set is convex when the line segment which joins any two points in it lies totally within the set.



Duality in LP

- Primal variables – quantities, dual variables – (imputed) prices
- Dual constraints – relationship between Marginal Revenue and Marginal Cost ($MC \geq MR$)

Duality

Assignment 5, problem 2

Primal - Maximize total revenue subject to resource availability:

Max $Zp = 8X1 + 10X2$

Subject to

$5X1 + 3X2 \le 15$ (capital)

$4X1 + 6X2 \le 24$ (labor)

X1 and X2 non-negative

X1 is production of footballs, and X2 is the production of basketballs

Dual – Minimize total cost subject to market equilibrium MC ≥ MR

1. Total cost = (total input uses) * (imputed prices) = (input supplies) * (imputed prices)

$TC = (5X1 + 3X2)\,Y1 + (4X1 + 6X2)\,Y2 =$

$= (5Y1 + 4Y2)\,X1 + (3Y1 + 6\,Y2)\,X2 =$

$= 15\,Y1 + 24\,Y2$

2. Marginal cost – first derivative of total cost with respect to output

$MC1 = \delta\,TC / \delta\,X1 = (5Y1 + 4Y2)$

$MC2 = \delta\,TC / \delta\,X2 = (3Y1 + 6\,Y2)$

3. Marginal revenue – first derivative of total revenue with respect to output

$MR1 = \delta\,Zp / \delta\,X1 = 8$;

$MR2 = \delta\,Zp / \delta\,X2 = 10$

4. Dual:

Min $TC = 15\,Y1 + 24\,Y2$

Subject to

$5Y1 + 4Y2 \ge 8$

$3Y1 + 6\,Y2 \ge 10$

Y1, Y2 – non-negative

Duality

Assignment 5, problem 1:

Primal – Minimize transportation cost

$$Min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\sum_{j=1}^{n} x_{ij} \le a_i,$$

$$\sum_{i=1}^{m} x_{ij} \ge b_j,$$

$$x_{ij} \ge 0$$

Dual – maximize value added subject to market equilibrium MC ≥ MR

Value added =

(total value of goods sold at destinations) –

(total value of goods bought at origins) =

$$\sum_{demand_j} p^d{}_j\, b_j - \sum_{\text{supply}_i} p^s{}_i\, a_i$$

MC = unit shipment cost; MR = difference in prices at demand and supply locations

$$p^d{}_j - p^s{}_i \le c_{ij}$$

(prices are non-negative)

Duality: *Optimization Results*

- Optimal values of decision variables – imputed prices
- Dual shadow price (marginal values for equations)
  - change in objective function for 1 unit change in RHS of constraint
  - equals the optimum value of decision variables in primal problem
- Dual slack / surplus (difference between level and RHS of equations)
  - difference between marginal costs and returns
  - equal reduced cost for non-basic decision variables in primal
- Dual reduced cost (marginal values for variables)
  - represents marginal costs of forcing nonbasic variable into the solution
  - equals slack / surplus in primal problem

GAMS:
SOLVE execution

- Model is translated into SOLVER language
- Comprehension aid is written to output
- Error check, if error – program termination
- SOLVER solves the model
- GAMS reports the status of the solution and load solution values from SOLVER

Comparison of Two GAMS Programs (*hw5, q1*)

- GENERATION TIME – time (in seconds) used since the syntax check finished. This includes the time spent in generating the model.
  - *0.016 versus 0.000*
- EXECUTION TIME – time (in seconds) used to execute actions such as data transformation, model solution, and report generation.
  - *0.016 versus 0.000*
- RESOURCE USAGE, LIMIT, gives information about the time, in seconds, taken by the solver and the upper limit allowed for the solver.
  - *0.000 versus 0.015*

http://www.gams.com/mccarl/speed.pdf - tips on how to shorten execution time

Excel Sensitivity Report

Range of coefficients in the objective function for which the values of variables in the optimal LP solution will remain unchanged

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B6	price, pd modesto	40	0	200	0	100
C6	price, pd sonoma	30	0	500	0	500
D6	price, pd fresno	80	0	300	0	100
F3	sacramento price, ps	0	0	-400	0	100
F4	oakland price, ps	0	0	-600	0	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B9	modesto, sacramento	40	100	40	0	10
B10	modesto, oaklan	40	100	40	10	0
B11	sonoma, sacramento	30	0	50	1E+30	20
B12	sonoma, oakland	30	500	30	20	30
B13	freson, sacramento	80	300	80	10	80
B14	fresno, oakland	80	0	90	1E+30	10

Shadow Price tells us how much the objective function will change if we change the right-hand side of the corresponding constraint within the limits given in the Allowable Increase/Decrease columns

Non-Linear Models in GAMS

GAMS Hints

- *Exponential: x**n*
  - calculated inside GAMS as *exp[n*log(x)]*.
  - not defined if *x* has a negative value (an error will result)
  - If the possibility of negative values for *x* is to be admitted *and* the exponent is known to be an integer, then use *power(x,n)*.

- *Alias (i,j)* - used to give another name to a previously declared set