The Transportation Problem

For ARE 521 QUANTITATIVE TECHNIQUES

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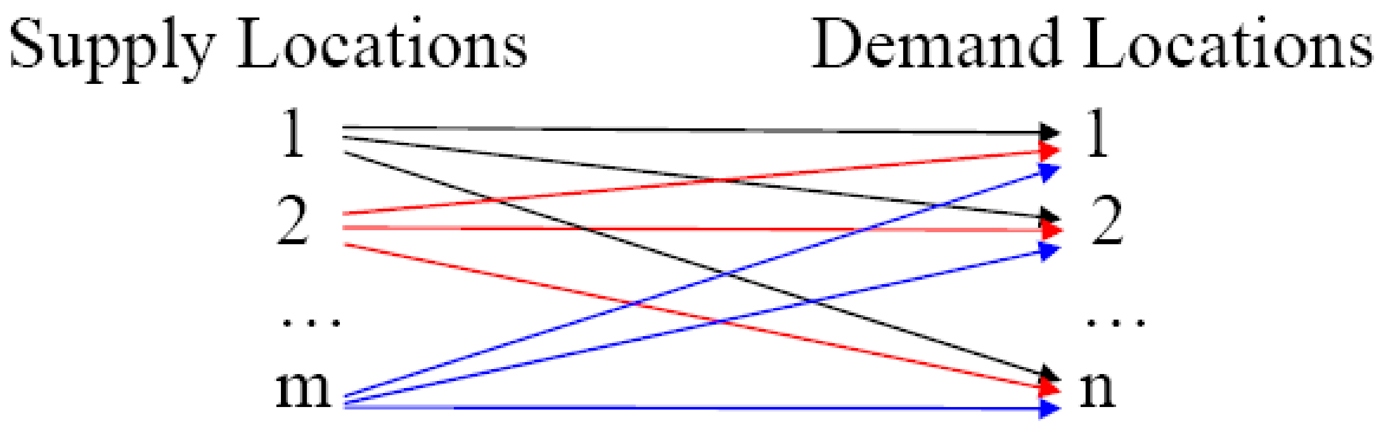
Transportation Problem

- This problem involves the shipment of a homogeneous product from a number of supply locations to a number of demand locations.



- Problem: given needs at the demand locations, how should we take limited supply at supply locations and move the goods. Further suppose we wish to minimize cost.

Based on McCarl 2005

Basic Concept

- Objective:
  - » Minimize cost
- Variables:
  - » Quantity of goods shipped from each supply point to each demand point
- Restrictions:
  - » Non negative shipments
  - » Supply availability at a supply point
  - » Demand need at a demand point

Based on McCarl 2005

Formulating the Problem: *Basic notation and the decision variable*

- We denote the supply locations as i
- We denote the demand locations as j
- Let us define our fundamental decision variable as the set of individual shipment quantities from each supply location to each demand location and denote this variable as $Move_{supply_i, demand_j}$

Based on McCarl 2005

Formulating the Problem: *The objective function*

- We want to minimize total shipping cost so we need an expression for shipping cost
- Let us define a data item per unit cost of shipments from each supply location to each demand location as $cost_{supply_i,demand_j}$
- Our objective then becomes to minimize the sum of the shipment costs over all $supply_i$, $demand_j$ pairs

$$\text{Minimize} \sum_{supply_i} \sum_{demand_j} cost_{supply_i,demand_j} \; Move_{supply_i,demand_j}$$

- which is the per unit cost of moving from each supply location to each demand location times the amount shipped summed over all possible shipment routes

Based on McCarl 2005

Formulating the Problem: *Constraints*

- the sum of outgoing shipments from the $supply_i$ supply point to all possible destinations ($demand_j$) to not exceed $supply_{supply_i}$ (*resource availability constraints*)

$$\sum_{demand_j} Move_{supply_i, demand_j} \leq supply_{supply_i}$$

- shipments into the $demand_j$ demand point should be greater than or equal to demand at that point. Incoming shipments include shipments from all possible supply points $supply_i$ to the $demand_j$ demand point (*minimum requirement constraints*).

$$\sum_{supply_i} Move_{supply_i, demand_j} \geq demand_{demand_j}$$

- nonnegative shipments

$$Move_{supply_i, demand_j} \geq 0$$

Based on McCarl 2005

$$\text{Minimize} \quad \sum_{\text{supplyi}} \sum_{\text{demandj}} \text{cost}_{\text{supplyi,demandj}} \text{Move}_{\text{supplyi,demandj}}$$

$$\text{s.t} \quad \sum_{\text{demandj}} \text{Move}_{\text{supplyi,demandj}} \leq \text{supply}_{\text{supplyi}} \quad \text{for all supplyi}$$

$$\sum_{\text{supplyi}} \text{Move}_{\text{supplyi,demandj}} \geq \text{demand}_{\text{demandj}} \quad \text{for all demandj}$$

$$\text{Move}_{\text{supplyi,demandj}} \geq 0 \quad \text{for all supplyi, demandj}$$

Based on McCarl 2005

Standard Form

$$\text{Minimize} \sum_{supplyi} \sum_{demandj} \text{cost}_{supplyi,demandj} \text{Move}_{supplyi,demandj} \quad \Rightarrow \quad Min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\text{s.t} \quad \sum_{demandj} \text{Move}_{supplyi,demandj} \leq \text{supply}_{supplyi} \quad \text{for all supplyi} \quad \Rightarrow \quad \sum_{j=1}^{n} x_{ij} \leq a_i ,$$

$$\sum_{supplyi} \text{Move}_{supplyi,demandj} \geq \text{demand}_{demandj} \quad \text{for all demandj} \quad \Rightarrow \quad \sum_{i=1}^{m} x_{ij} \geq b_j ,$$

$$\text{Move}_{supplyi,demandj} \geq 0 \quad \text{for all supplyi, demandj} \quad \Rightarrow \quad x_{ij} \geq 0$$

where

i – supply locations, *j* – demand locations,

cij – unit shipping cost, *xij* – shipment volumes,

bj – demand levels, *ai* – supply limits

Example

A trucking company has contracted to supply weekly three supermarkets located in Modesto, Sonoma, and Fresno, California, with potatoes kept at two warehouses in Sacramento and Oakland, also in California. The weekly availability (supply) of potatoes at the warehouses is 400 and 600 tons, respectively. The supermarkets’ weekly needs (demands) are 200, 500, and 300 tones of potatoes, respectively. (the transportation unit costs are given below.)

The objective is to find supply routs that minimizes the total transportation cost of the trucking firm. The contract stipulates that all supermarkets must receive at least the quantity of potatoes specified above.

Based on Paris 1991

Example (cont.)

- Two warehouses: Sacramento and Oakland
- Three supermarkets: Modesto, Sonoma, and Fresno
- Quantity availability:
  - Supply available

Sacramento	400 tones
Oakland	600 tones

  - Demand required

Modesto	200
Sonoma	500
Fresno	300

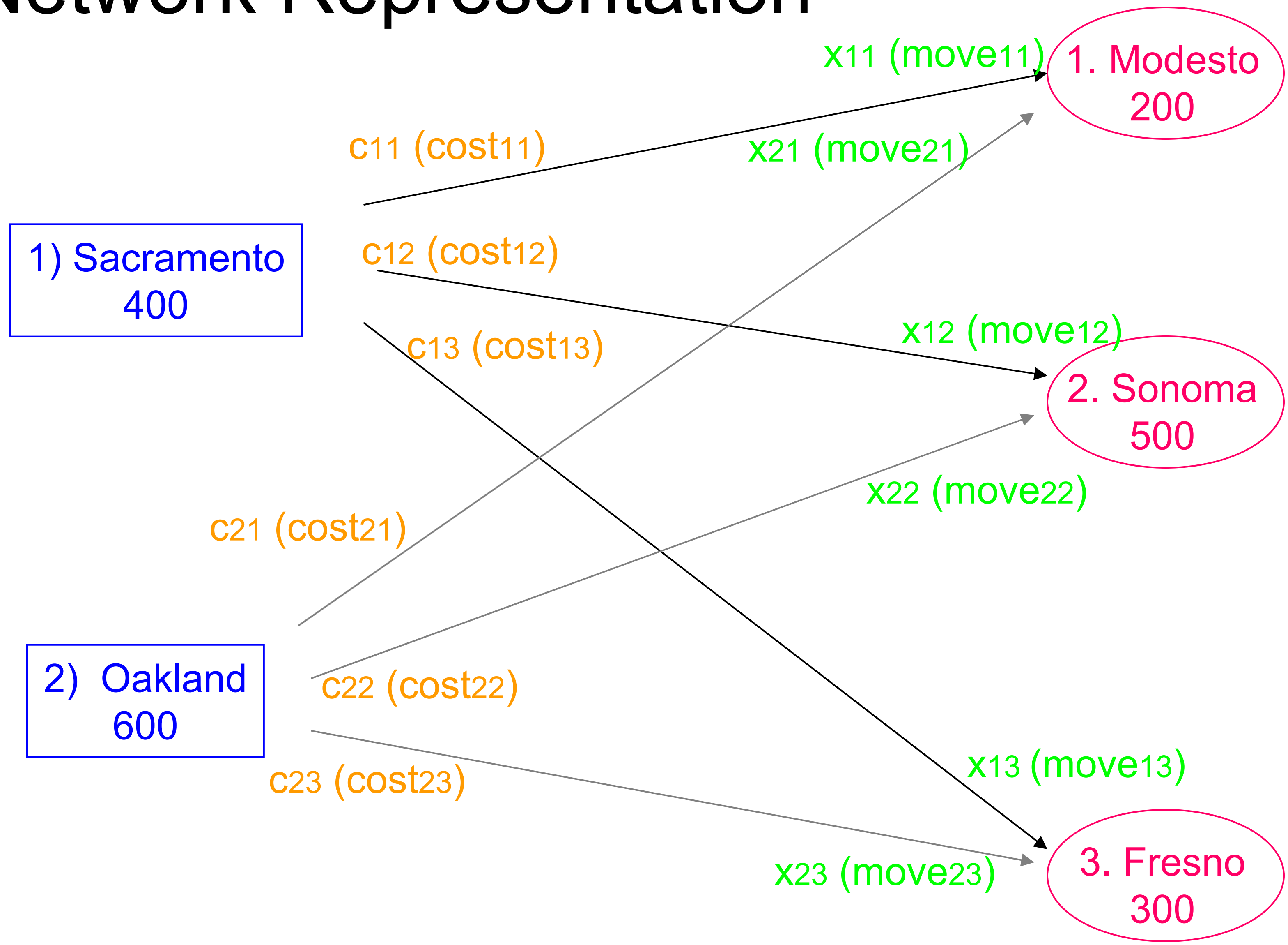
Based on Paris 1991

Example (cont.): *Transportation Unit Cost (dollars per ton)*

warehouse	Supermarkets		
	Modesto	Sonoma	Fresno
Sacramento	40	50	80
Oakland	40	30	90

Based on Paris 1991

Network Representation



Based on Paris 1991

Example:
Problem Formulation

- Objective function:
  - minimize total cost of shipment

Min $cost_{11}move_{11}$ + $cost_{12}move_{12}$ + $cost_{13}move_{13}$ + $cost_{21}move_{21}$ + $cost_{22}move_{22}$ + $cost_{23}move_{23}$

$$\Rightarrow \quad MinTC = \sum_i \sum_j \mathrm{cost}_{ij} move_{ij}$$

- Demand constraint

$move_{11} + move_{21} \geq demand_1$

$move_{12} + move_{22} \geq demand_2$

$move_{13} + move_{23} \geq demand_3$

$$\Rightarrow \quad \sum_i move_{ij} \geq demand_j,$$

for all *j*

Example: *Problem Formulation* (cont.)

- Supply constraints

$move_{11} + move_{12} + move_{13} \leq supply_1$

$move_{21} + move_{22} + move_{23} \leq supply_2$

$$\sum_j move_{ij} \leq \text{supply}_i, \qquad \text{for all } i$$

$$-\sum_j move_{ij} \geq -\text{supply}_i, \qquad \text{for all } i$$

Example: *Tableau*

move11 (x11)	move12 (x12)	move13 (x13)	move21 (x21)	move22 (x22)	move23 (x23)		
40	50	80	40	30	90		min
1			1			≥	200
	1			1		≥	500
		1			1	≥	300
-1	-1	-1				≥	-400
			-1	-1	-1	≥	-600

Example: *Optimal Shipping Pattern*

	Destination					
	Modesto		Sonoma		Fresno	
	Units	Variable	Units	Variable	Units	Variable
Sacramento	100	move11 (x11)			300	move13 (x13)
Oakland	100	move21 (x21)	500	move22 (x22)		

Example: *Optimal Solution*

- Objective value $47,000

Variable	*Value*	*Reduced cost*
Move11 (x11)	100	0
Move12 (x12)	0	20
Move13 (x13)	300	0
Move21 (x21)	100	0
Move22 (x22)	500	0
Move23 (x23)	0	10

Equation	*Slack*	*Shadow price*
Supply1	0	Eps
Supply2	0	0
Demand1	0	40
Demand2	0	30
Demand3	0	80

Example: *Solution*

- shadow price represents marginal values of the resources, e.g.
  - marginal value of additional units at supply locations are (close to) zero
  - marginal value of unit decrease in demand at demand locations = unit transportation cost to that location
- reduced cost represents marginal costs of forcing nonbasic variable into the solution

Based on McCarl 2005

Example: *Reduced Cost*

- Assume the trucking company is required to ship 1 tone of potatoes from Sacramento to Sonoma
  - Shipment from Sacramento to Sonoma cost $50
  - The trucking company also decreases supply from Oakland to Sonoma (saving of $30)
  - To stay within supply limits, the company decreases supply from Sacramento to Modesto (saving of $40)
  - To meet demand constraints, the company increases supply from Oakland to Modesto (cost $40)
  - Resulting change in transportation cost = reduced cost =

    $50 - $30 + $40 - $40 = $20

Transportation Model: *Assumptions*

- Transportation cost are independent of volume
- Supply and demand are known and independent on price charged for the product
- Unlimited capacity to ship across any particular transportation route
- Single commodity

Transportation Model: *Important Extensions*

- Transshipment
  - Transshipment through intermediate warehouses is permitted
- Spatial equilibrium
  - Quantities supplied and demanded depend on prices

GAMS-IDE

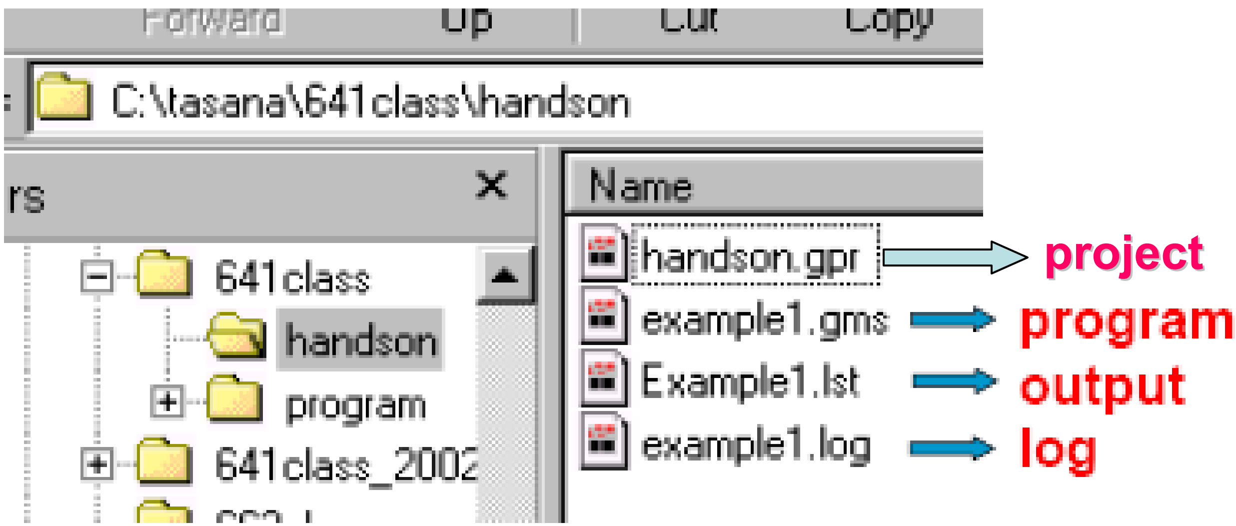
- GAMSIDE
  - GAMS – Generalized Algebraic Modeling System
  - IDE – Integrated Development Environment,

    A Windows graphical interface to run GAMS

- Using GAMSIDE
  - Open the IDE through the icon
  - Create a project
  - Create or open an existing file of GAMS instructions

Courtesy of B.A.McCarl Apr.2000

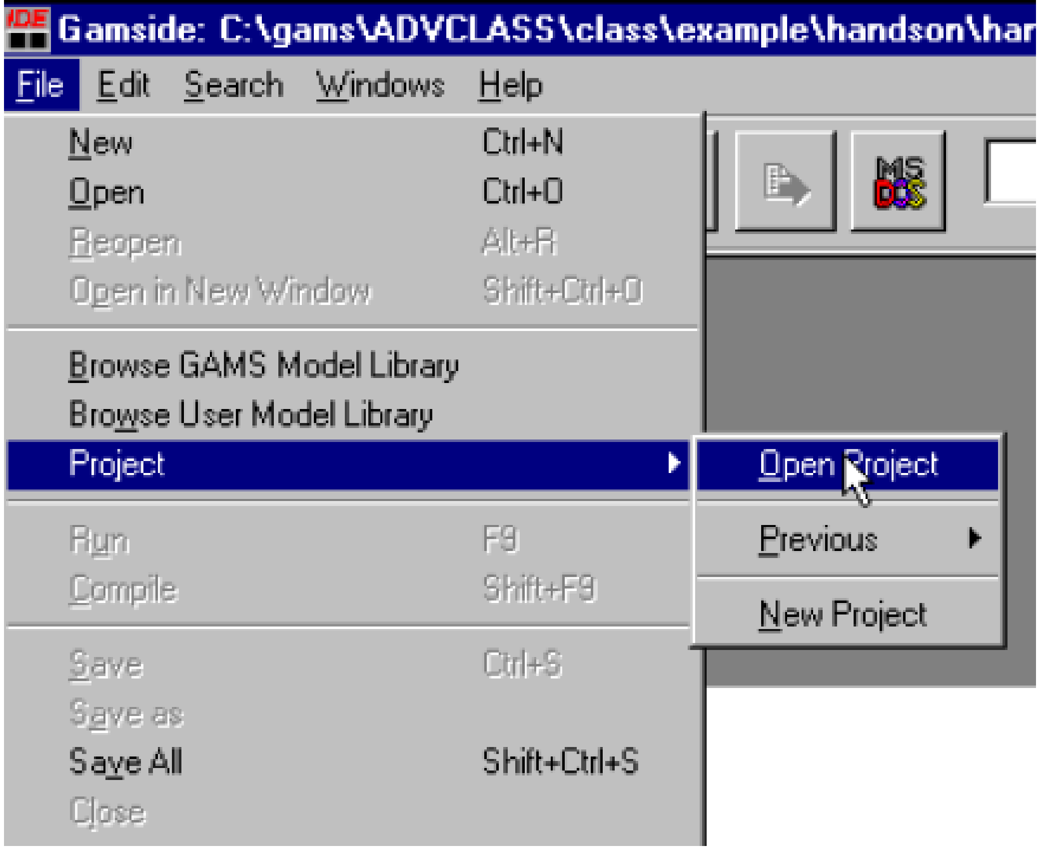
GAMSIDE Files



Based on McCarl and Gillig

What is a Project?

- The project location determines where all saved files are placed (to place files elsewhere use the save as dialogue)

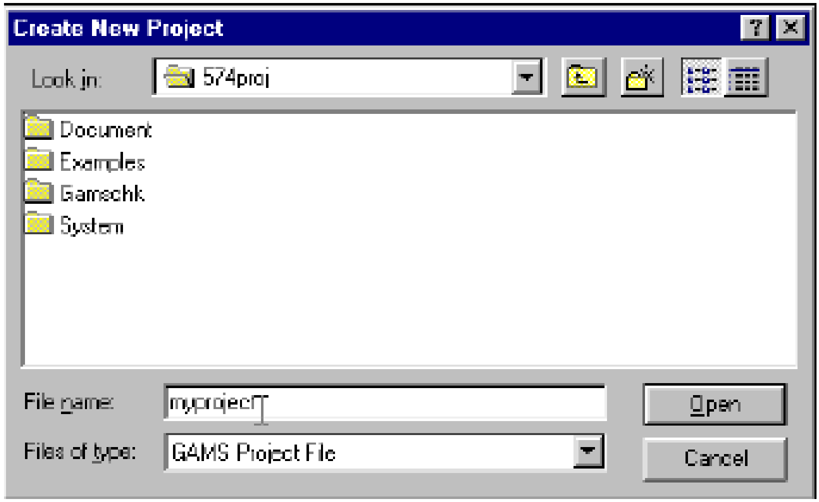


- The project saves file names and program options associated with the effort.

- It is recommended to define a new project every time you wish to change the file storage directory.

Courtesy of B.A.McCarl Apr.2000

GAMSIDE Project



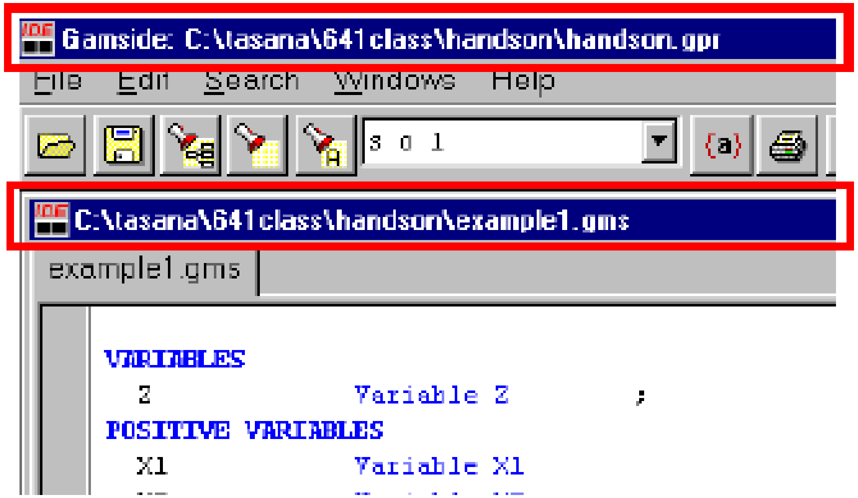
- Define a project name and location. Put it in a directory you want to use. All files associated with this project will be saved in that directory.

- In the “File name” area type in a name for the project file you wish to use. Save the file.

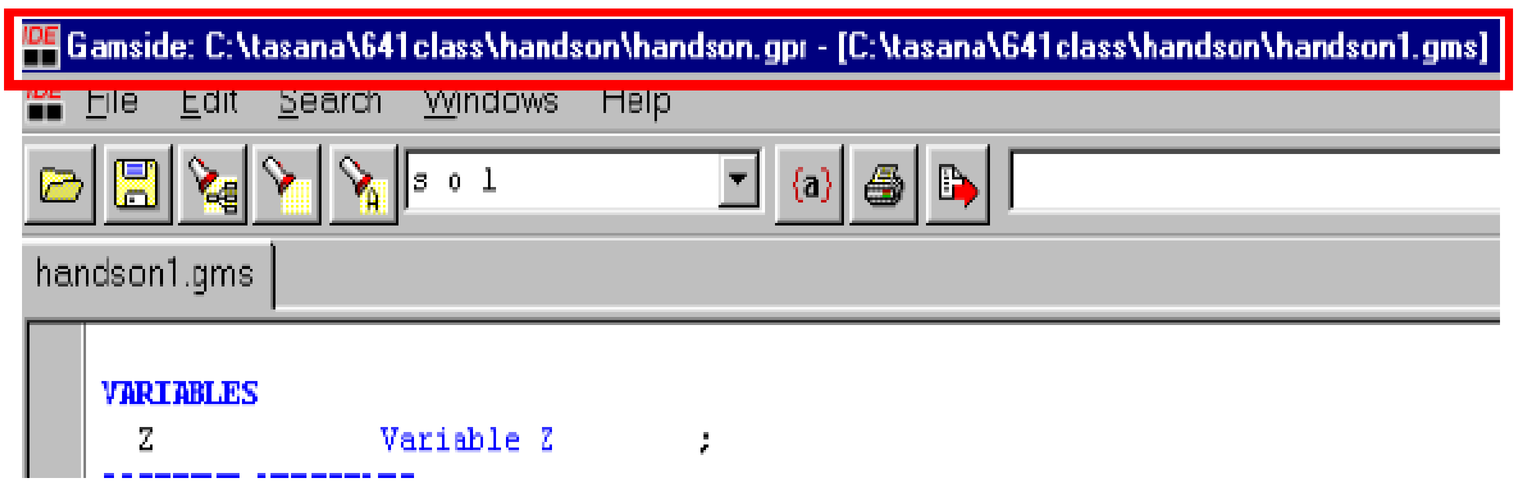
- File name extension gpr stands for GAMS project.

Courtesy of B.A.McCarl Apr.2000

GAMS IDE - Cautions on PROJECT



Things always work best when the project file points to the directory where the files you want to work on are stored. This involves contrasting the path of the project and the files. Here, there is no problem since the project and the program file are the same directory.

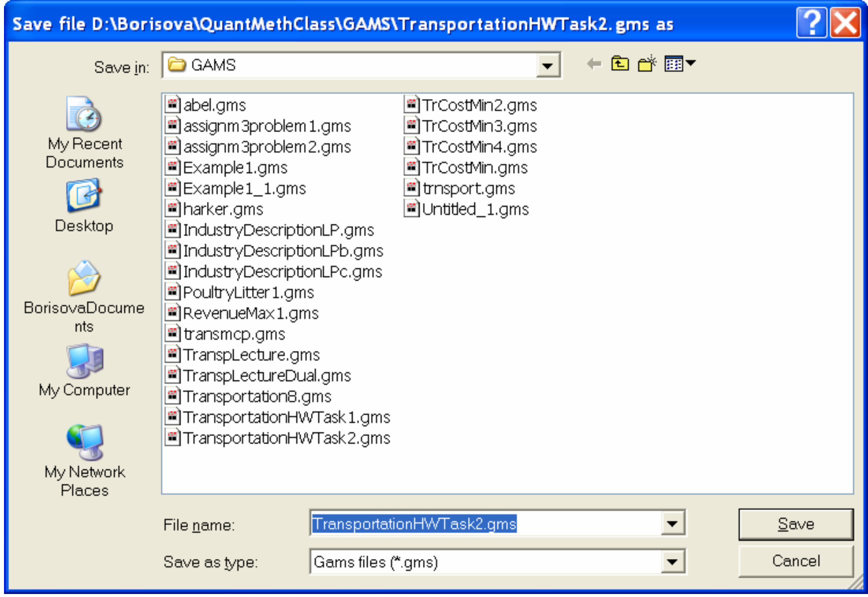



```
**** FILE SUMMARY

INPUT      C:\TASANA\641CLASS\PROGRAM\TASANA\HANDSON1.GMS
OUTPUT     C:\TASANA\641CLASS\PROGRAM\TASANA\HANDSON1.LST
```

Slide is taken from McCarl and Gillig

Opening or Creating a File with GAMS Instructions

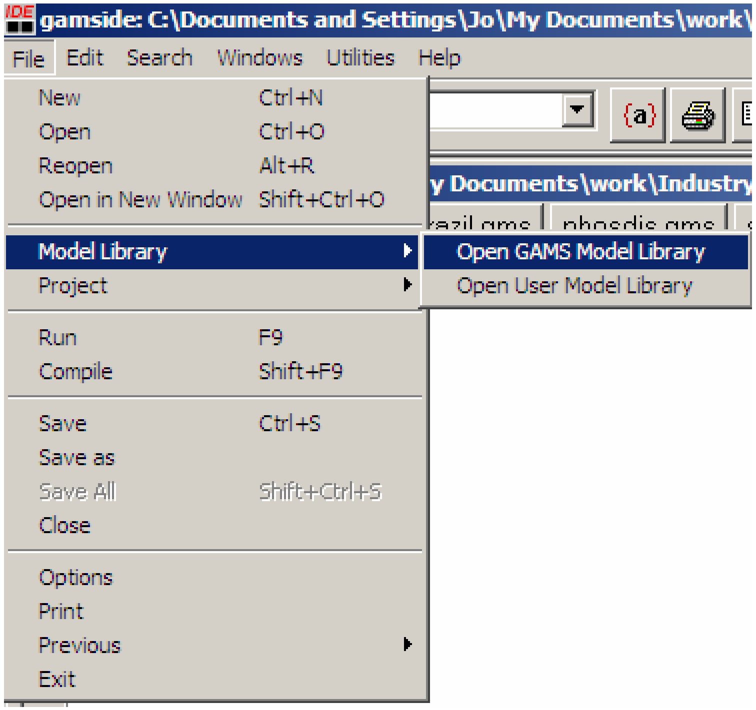


- You can
  - Create a new file
  - Open an existing file
  - Open a model library file

- File name extension – .gms

- GAMSIDE recognizes codes (and adds color differentiation) only in the files *saved* as .gms files.

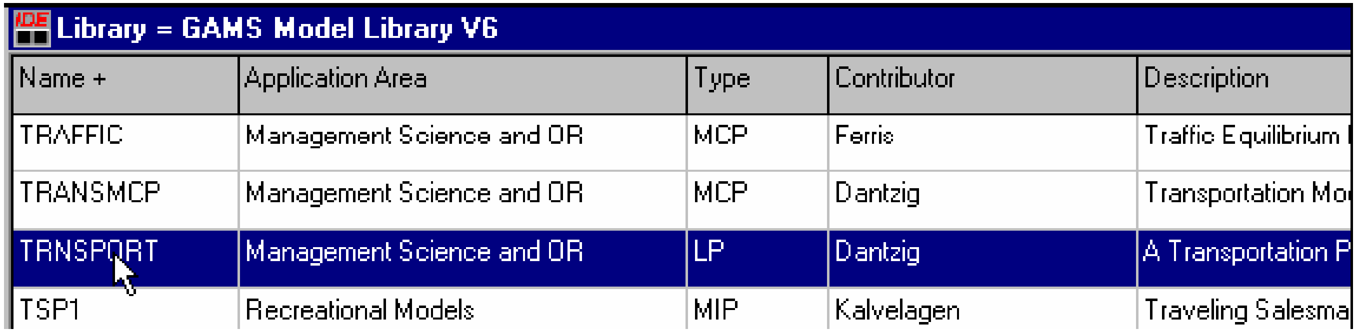
GAMS Model Library



- collected models to be used as of examples

- Standard textbook examples to illustrate problem formulation / GAMS features

- models that have been used in policy or sector analysis and are interesting for both the methods and the data they use



Library = GAMS Model Library V6

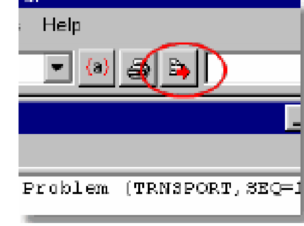
Name +	Application Area	Type	Contributor	Description
TRAFFIC	Management Science and OR	MCP	Ferris	Traffic Equilibrium
TRANSMCP	Management Science and OR	MCP	Dantzig	Transportation Mo
TRNSPORT	Management Science and OR	LP	Dantzig	A Transportation P
TSP1	Recreational Models	MIP	Kalvelagen	Traveling Salesma

Output and Log files

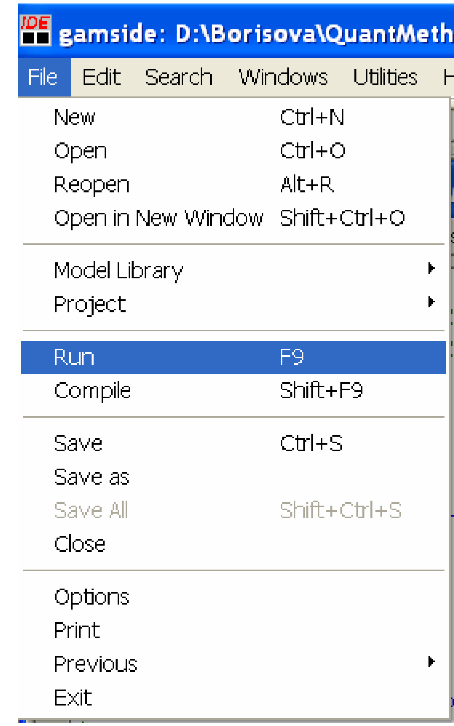
- Log file
  - Located in the same directory as the project file
  - File extension - .log
  - Contain whatever GAMS-IDE shows during the run (content of *Process window*)
  - Clicking on any of the **black-colored line** will activate the *.lst* output file
    - Clicking “Reading Solution for model” open the .lst file and position the window at the SOLVE SUMMARY
  - Clicking on the red-colored line will cause the cursor to jump in the .gms file to the line with error

- Output (listing) file
  - Located in the same directory as the project file
  - File name extension - .lst
  - Stores results of the model run

Running GAMS programs



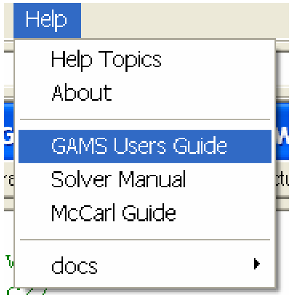
- click on the run button, or
- use *File* menu, or
- press the F9 key



- Note
  - Option “Compile” – performs initial check of the program
  - Option “Run” – check the program, and (if there is no error) run optimization and generate output

GAMS Help

- GAMS Users Guide
- McCarl Guide
- Solver Manual (discussion of various GAMS Solvers)
- GAMS documents – additional materials on GAMS, IDE, Solvers



Useful GAMS Websites

- Download GAMS student version
  http://www.gams.com/download/
- GAMS development corporation
  http://www.gams.com/
- Bruce McCarl's web-site
  http://agecon2.tamu.edu/people/faculty/mccarl-bruce/

References

- McCarl, B. Course Materials from GAMS 2 Class. http://www.gams.com/mccarl/useide.pdf

- McCarl B. 2005. Basic LP problem formulations
- http://agecon2.tamu.edu/people/faculty/mccarl-bruce/622class/overhead05basiclpformulate.pdf

- McCarl's and Spreen's book at http://agecon2.tamu.edu/people/faculty/mccarl-bruce/books.htm

- McCarl B & D Gillig. Introduction to GAMS IDE. http://agecon2.tamu.edu/people/faculty/mccarl-bruce/641clas/02_641_intro_gamsIDE_.pdf

- Paris 1991. An Economic Interpretation of Linear Programming. Iowa State University Press / Ames