ARE 521
Wednesday Oct 19
Goal Programming

Guest Lecturer – Mike Strager, Ph.D.

Todays guest lecture outline

- Overview of goal programming
- Basic approaches (Weighted Goal Programming, Chebyshev Goal Programming, Lexicographic Goal Programming, Compromise Programming)
- Weight solicitation
- Use of fuzzy sets and membership functions
- Advantages/disadvantages of approaches
- Future directions and opportunities

Goal Programming (GP)

- A form of linear programming that allows for consideration of multiple goals
- GP can be used to determine the optimal solution to a multi-objective decision problem

What is multiple objective decision making?

- A form of decision analysis that seeks to analyze complex decision problems by dividing the problem into smaller understandable parts
- Then, we integrate the parts in a logical manner to produce a meaningful solution

Multiple objectives in decision making

- The old saying that still holds in everyday life is that human beings "want their cake and to eat it too."
  - we want both quality and quantity
  - higher incomes and more free time
  - we need profits as well as social expenditure
  - low inflation as well as high employment, and so on

Generally speaking….

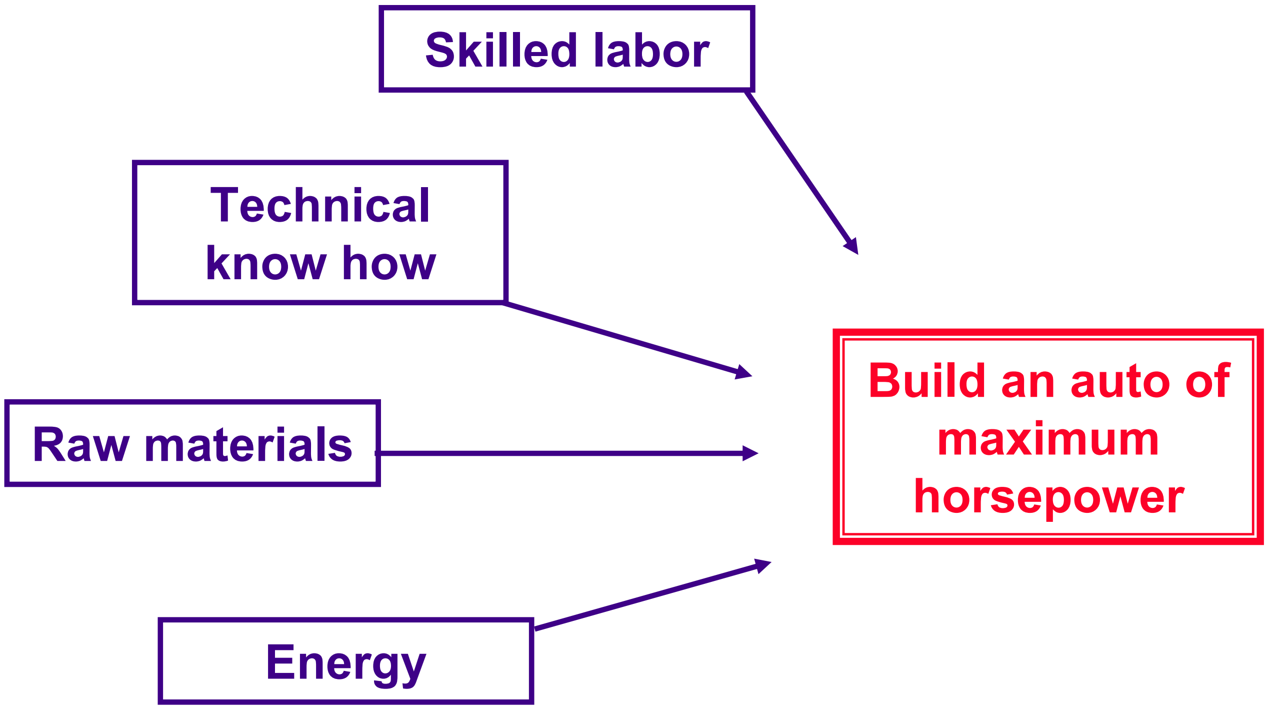
- Individuals, groups, and organizations, in their decision making efforts,
  - pursue multiple objectives
  - set multiple goals
  - evaluate their options according to multiple criteria and as a consequence experience conflict.
- Decision making under these such conditions is characterized by incessant attempts at conflict resolution and the simultaneous attainment of goals.

Milton Friedman quote:

- To emphasize the role of multi objective framework of thought in economics he said:

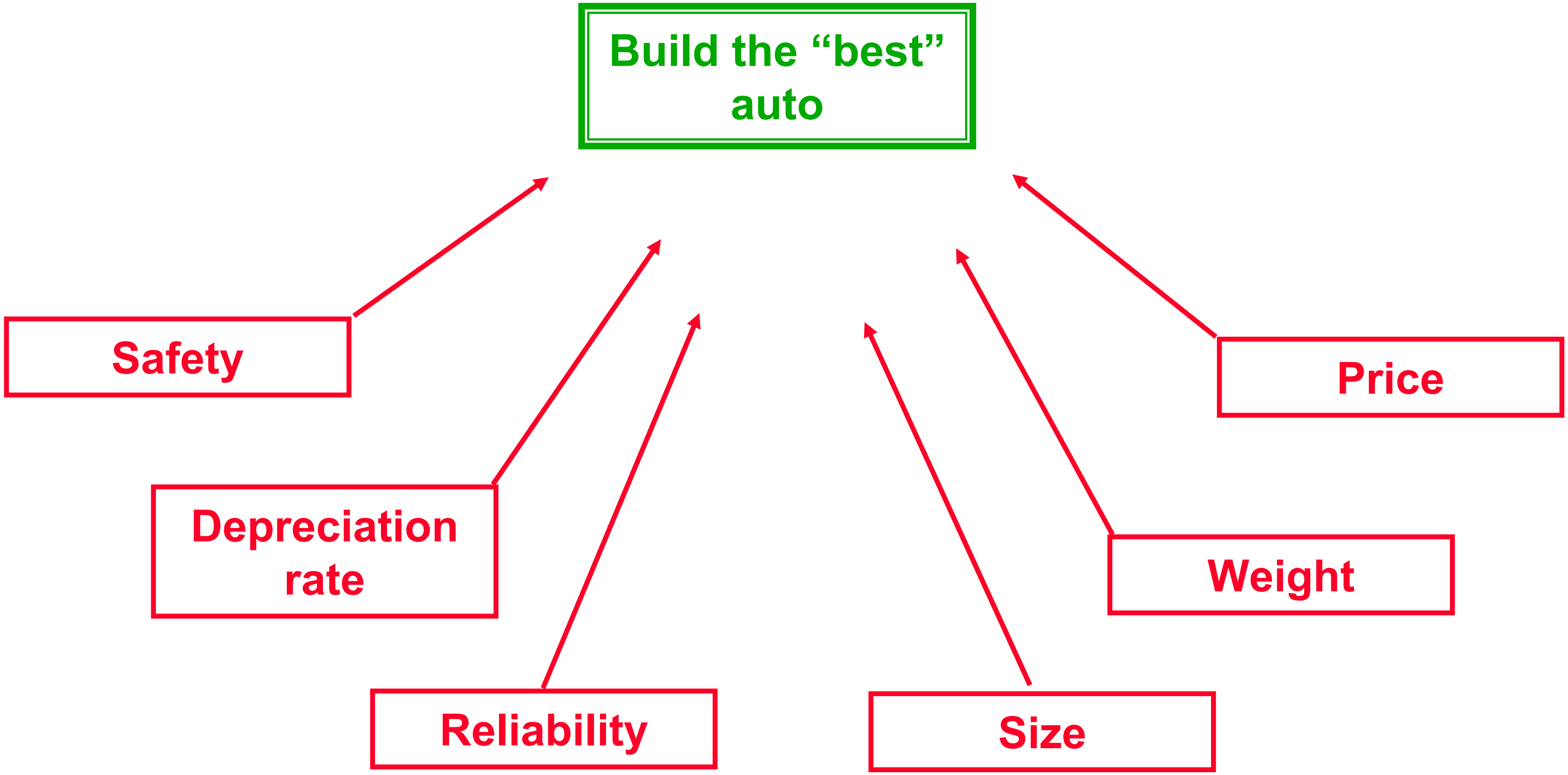
> "An economic problem exists whenever scarce means are used to satisfy alternative ends. If the means are not scarce, there is no problem at all. If the means are scarce but there is only a single end, the problem of how to use the means is a technological problem. No value judgments enter into its solution; only knowledge of physical and technical relationships." (Zeleny, 1982, p26)

Illustration (single objective decision making)



"a purely technical problem"

Illustration (multiple objective decision making)

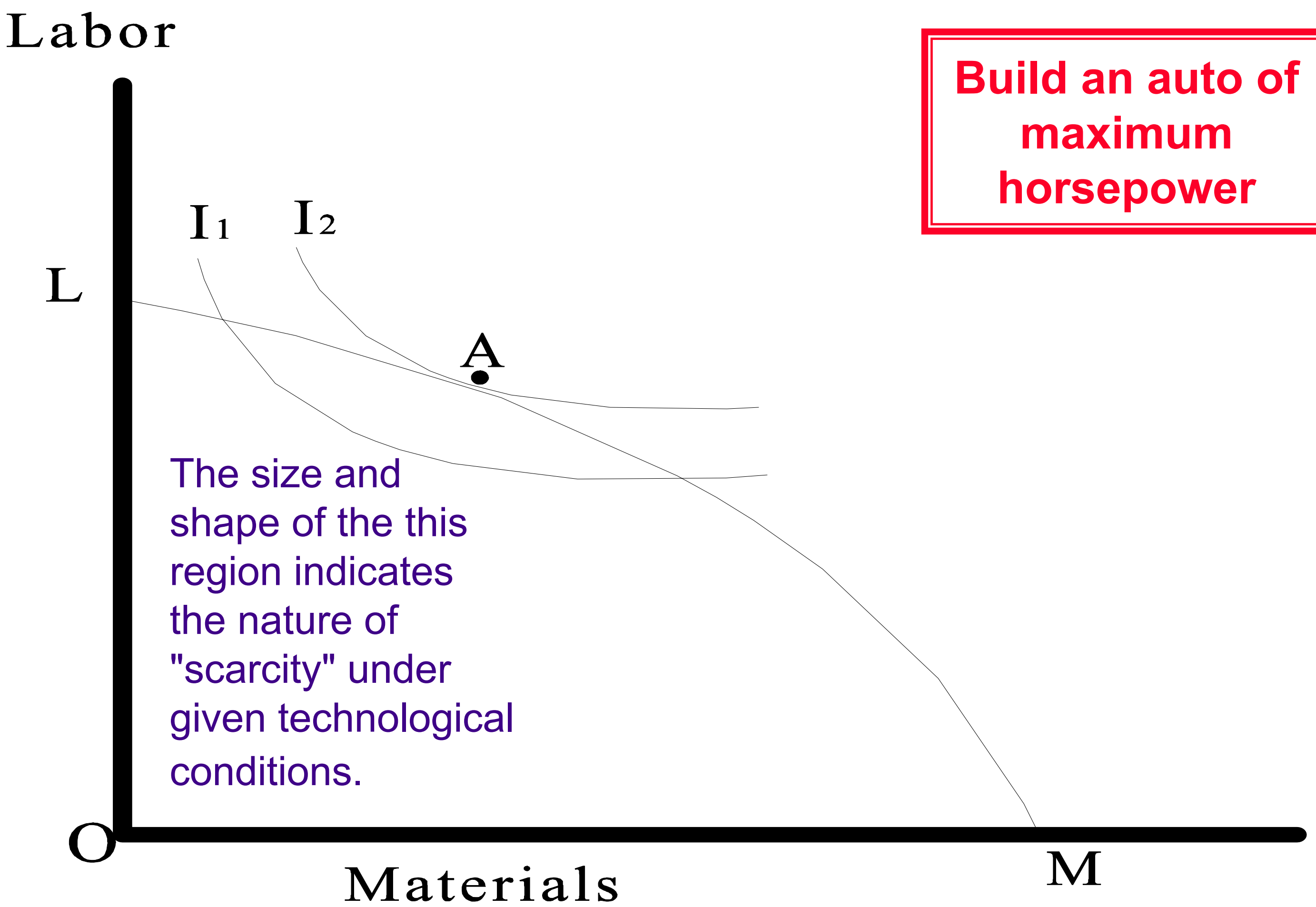


No single end or no single criterion, a multiple criteria decision problem or an economic problem according to Freidman’s definition

Human value judgments, trade off evaluations, and assessments of criteria now become integral to the problem

Graphically (single objective decision making)

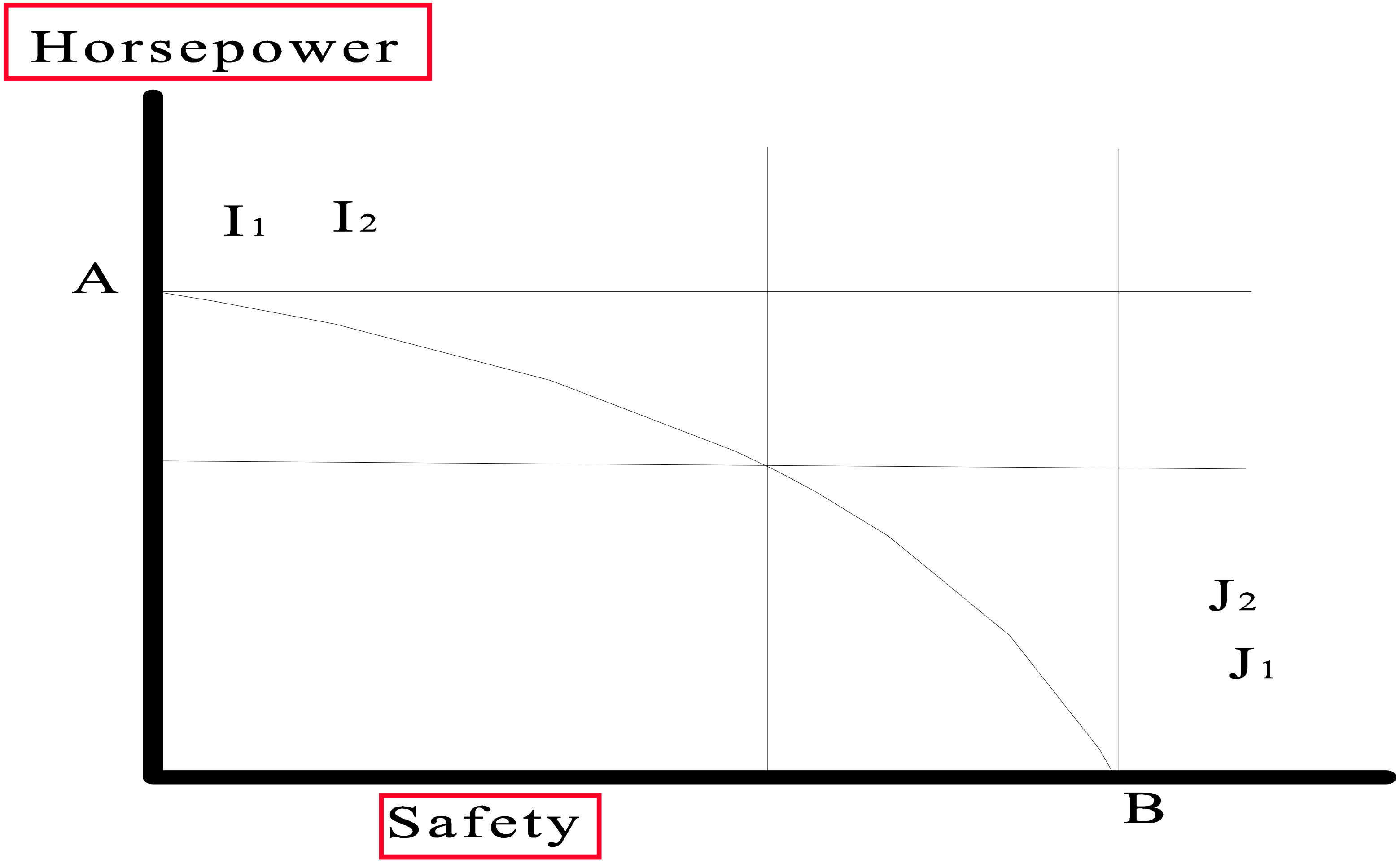
- **Technological problem**



Build an auto of maximum horsepower

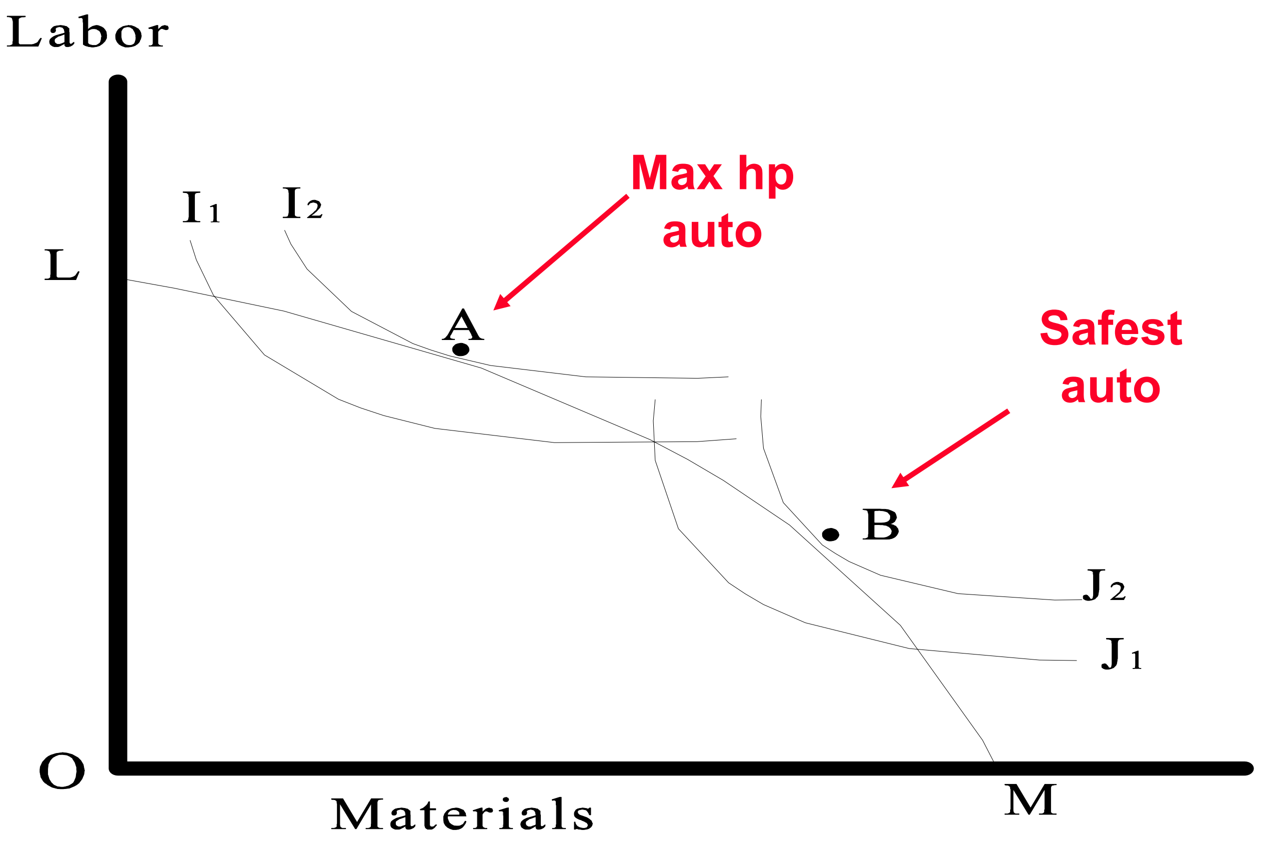
Graphically (multi objective decision making)

- Economic problem in terms of inputs



Graphically (multi objective decision making)

- Economic problem in terms of outputs



Note

- Traditional economics does not perceive problems in terms of alternative ends (despite Friedman's definition quoted earlier).
- Rather, it usually conceives of problems in terms of single objectives such as utility maximization in decision-making organizations and profit maximization or cost minimization in economic organizations.
- In this sense only purely technological problems are dealt within economics (Zeleny, 1982).

Recognition of MODM

- occurred because of weaknesses with the conventional mathematical programming models in decision making problems.
- Mathematical programming models first provide feasible solutions which satisfy the constraints of the problem and then order the feasible solutions according to an objective function representing the preferences of the decision maker.
- The optimal solution, found from the feasible set, is the highest possible value for the objective function (Romero and Rehman, 1989).

Problem with that approach

- Romero and Rehman (1989) note that the decision maker is usually not interested in ordering the feasible set according to just a single criterion but strives to find an optimal compromise among several objectives.
- Most decision makers are confronted with multiple, sometimes simultaneous, objectives.
- Another problem with using a single objective linear programming procedure lies in the restriction of commensurable units in determining an optimal solution (Keeney and Raiffa, 1993).
- Furthermore, a linear programming model implies proportionality, additivity of costs, and the effects of resource uses which, in most cases, are not true (Tecle and Yitayew, 1990).

More MODM notes

- In a multiple criterion or economic problem human value judgments, tradeoff evaluations, and assessments of the importance of criteria are an integral part of the problem.
- MODM is a positive science in its use as compared to normative. It is positive because it attempts to explain "what is."
- The only likeness to a normative science is in collecting criteria weights from the preferences of individuals.
- There is some tie here to utility theory in how to capture individual preferences but overall MCDM is not normative like the profit maximization framework or cost minimization where the emphasis is placed on "what should be."

Difference between GP and LP

- Where LP identifies from the set of feasible solutions, the point that optimizes a single objective…..
- GP determines the point that best satisfies the set of goals in the decision problem
- GP attempts to *minimize* the deviations from the goals

GP approach

- **Requires the decision maker to specify the most desirable value (goal) for each objective as the aspiration level**
- **The objective functions are then transferred into goals:**

$$f_k(x) + d^-_k - d^+_k = a_k \qquad \text{for } k = 1, 2, \ldots\ldots, q$$

$$d^-_k,\ d^+_k >= 0,\ (d^-_k,\ d^+_k) = 0$$

where a_k is the aspiration level for the kth objective, and d^-_k and d^+_k are negative and positive goal deviations, respectively, that is, nonnegative state variables that measure deviations of the current value of the kth criterion function from the corresponding aspiration level.

GP notes

- Two types of variables are part of any goal programming formulation…

  The decision variables, x

  and

  The deviation variables, d

- The objective function in a goal programming problem is always minimized and must be composed of deviation variables only (decision variables are implicitly assigned coefficients of zero)

- An optimal solution is then understood as one that minimizes the deviations from the aspiration levels!!!!!!!!!!!!!!

Three basic approaches to GP

- Weighted Goal Programming
- Chebyshev Goal Programming
- Lexiographic Goal Programming

- NOTE that these three are also know as minisum, minimax, and preemptive priority goal programming respectively

Weighted goal programming (WGP)

- **The objective is to find a solution that minimizes the weighted sum of the goal deviations**
- **The objective function can be expressed as:**

$$\text{Min} \sum_k (w^-_k d^-_k + w^+_k d^+_k)$$

Where w^-_k and w^+_k are weights corresponding to several goal deviations.

WGP notes

- The weights in a WGP represent additional information reflecting the decision maker's preferences with respect to the deviation variables.
- WGP assumes that the positive and negative deviations of the criterion outcomes are equally undesirable

That is, that decision maker perceives both overachievement and underachievement of specified goals as equally undesirable outcomes.

The decision maker behaves according to a strictly satisficing principle

Methods to create weights for criteria

- Point allocation
- Rank sum
- Rank reciprocal
- Rank exponent
- Pairwise comparison

Point allocation

- Weights are estimated by the decision maker on a predetermined scale, for example 0 to 100.
- In this approach, the more points a criterion receives, the greater its relative importance.
- The total of all criterion weights must sum to 100. This method is easy to normalize.

	Weight
Elev change	10
Wetlands	25
Roads	5
Row crops	40
Mining	20

Total	100

Rank sum, reciprocal, exponent

Ranking methods to assign weights, after Malczewski (1999)

		Rank Sum		Rank Reciprocal		Rank Exponent	
		Weight	Normalized	Weight	Normalized	Weight	Normalized
Criterion	Straight rank	$(n-r_j+1)$		$(1/r_j)$		$(n-r_j+1)^p$ p=2	
Elev change	4	2	.133	.250	.109	4	.073
Mining	2	4	.267	.500	.219	16	.291
Roads	5	1	.067	.200	.088	1	.018
Wetlands	1	5	.333	1.000	.438	25	.454
Row crops	3	3	.200	.333	.146	9	.164
		15	1.000	2.283	1.000	55	1.000

Straight rank is first step

n = total number of criteria
r_j = the straight rank

Notes on rank sum, reciprocal, exponent

- These three ranking methods are very attractive due to their simplicity.
- They also provide a satisfactory approach to weight assessment. As a starting point in deriving weights, the three ranking methods provide a way to simplify multicriteria analysis.
- However, they are limited by the number of criteria to be ranked.
- This method is really not appropriate for a large number of criteria since it becomes very difficult to straight rank as a first step.
- Another problem is in the lack of any real theoretical foundation.

Pairwise comparison

Intensity	Definition
1	Equal importance
2	Equal to moderate importance
3	Moderate importance
4	Moderate to strong importance
5	Strong importance
6	Strong to very strong importance
7	Very strong importance
8	Very to extremely strong importance
9	Extreme importance

Pairwise comparison method weight calculation

	Step 1			Step 2			Weights
	roads	wetlnd	mining	roads	wetlnd	minin	
roads	1	4	7	.718	.769	.538	(.718+.769+.538)/3=.675
wetlands	1/4	1	5	.179	.192	.385	(.179+.192+.385)/3=.252
mining	1/7	1/5	1	.102	.039	.077	(.102+.039+.077)/3=.073
	1.393	5.200	13.0	1.000	1.000	1.00	1.000

Example of pc sheet

Name____

For each paired choice below, fill in the circle to indicate which criteria is more important (or equal) for ranking watersheds for pollution potential

criteria	strongly prefer	prefer	somewhat prefer	equal	somewhat prefer	prefer	strongly prefer	criteria
elevation change	O	O	O	O	O	O	O	wetlands
wetlands	O	O	O	O	O	O	O	road length
road length	O	O	O	O	O	O	O	row crops
row crops	O	O	O	O	O	O	O	mining area

Notes on pairwise comparison

- Advantages of pairwise comparison include:
  - the method requires only two criteria to be considered at one time,
  - and the method has been tested theoretically and empirically for a variety of decision situations including spatial decision making (Malczewski, 1999)

Chebyshev goal programming (CGP)

- Can be considered a specific form of a weighed goal programming approach
- It seeks the solution that minimizes the worst unwanted deviation from any single goal

CGP notes

- Instead of using subjective notions to set the aspiration levels for the objectives, a set of single optimization problems is solved to arrive at the "best" and "worst" possible values of each objective.
- The best values are then used as aspiration levels for the objectives.
- The objective then becomes to minimize the deviation from those aspiration levels so that the worst deviation from any single-goal aspiration level is minimized.

NOTE: CGP can also be formulated as fuzzy goal programming

Fuzziness

- Refers to vagueness and uncertainty, in particular to the vagueness related to human language and thinking
  - “ the set of tall people”
  - “all people living close to my home”
  - “all areas that are very suitable for corn”
- Provides a way to obtain conclusions from vague, ambiguous or imprecise information. It imitates the human reasoning process of working with non precise data.

Crisp sets versus fuzzy sets

Crisp sets
Characteristic function

$X_A: X \rightarrow \{0,1\}$

where

$$X_A(X) = \begin{cases} 1 \text{ iff } X \in A \\ 0 \text{ iff } X \notin A \end{cases}$$

Fuzzy sets
Membership function

UA: X -> [0, 1]

where

$U_A(X)$ is the membership value of x in A

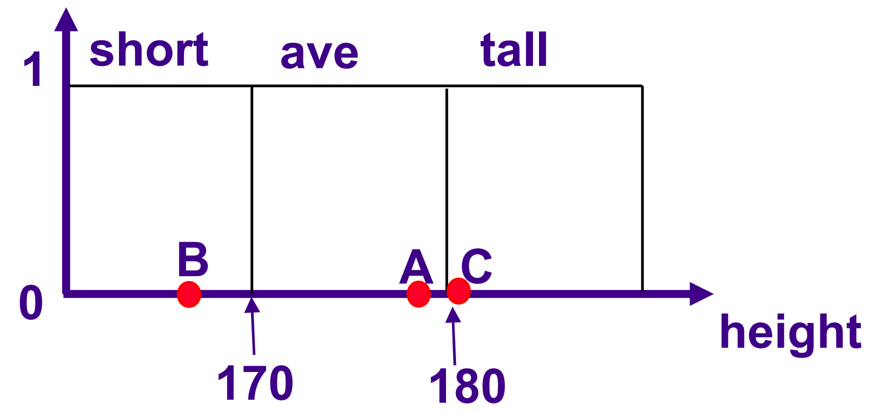
Example of crisp sets versus fuzzy sets

- Height of three adults: A is 178cm, B is 166cm, and C is 181cm

Crisp set

short	under 170cm
average	170 to 180cm
tall	over 180cm

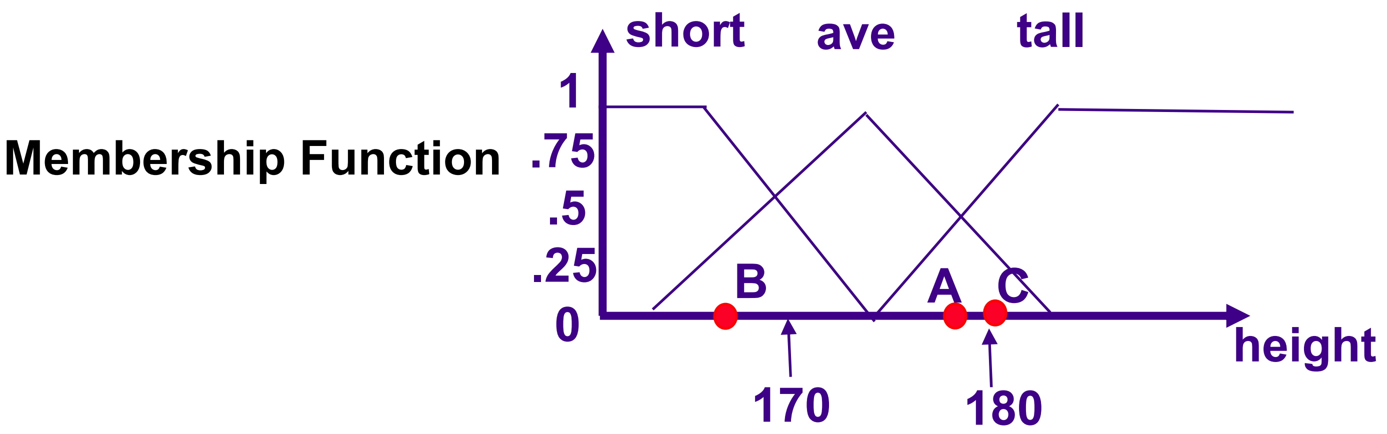
Characteristic Function



	short	ave	tall
A	0	1	0
B	1	0	0
C	0	0	1

Example of crisp sets versus fuzzy sets

- Height of same three adults: A is 178cm, B is 166cm, and C is 181cm



	short	ave	tall
A	0	.4	.6
B	.8	.2	0
C	0	.3	.7

Choice of membership function

- The grade of membership should be 1 at the center of the set
- The membership function should fall off in an appropriate way from the center through the boundary
- The point with membership grade .5 (crossover point) is at the boundary of the crisp set

Choice of membership function

- The membership function depends on the application
- EX: moderate elevation may be defined much differently in West Virginia versus moderate elevation in Colorado
- There are linear and sinusoidal membership functions, I will focus on linear in the example which follows

Example

- We want to find an ideal location for building a ski resort based on the following criteria:
  - Steeply sloped terrain
  - North facing hillside
  - Higher elevation
  - Near water

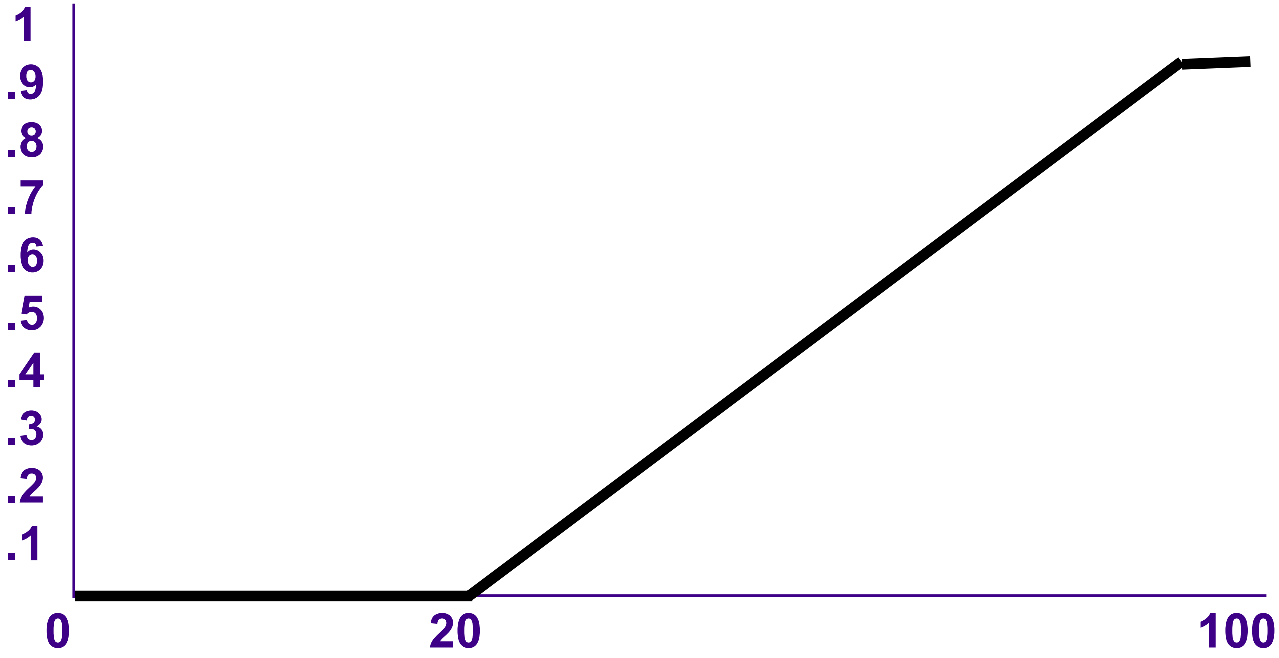
Fuzzy concepts

- Slope is favorable when it is above 20 percent
- Aspect if favorable when the terrain is oriented toward the North
- At this location in WV, preferred elevation is 380 to 800 meters
- It is favorable to be within 3600 meters of water

Slope membership function

"Slope is favorable when it is above 20 percent"

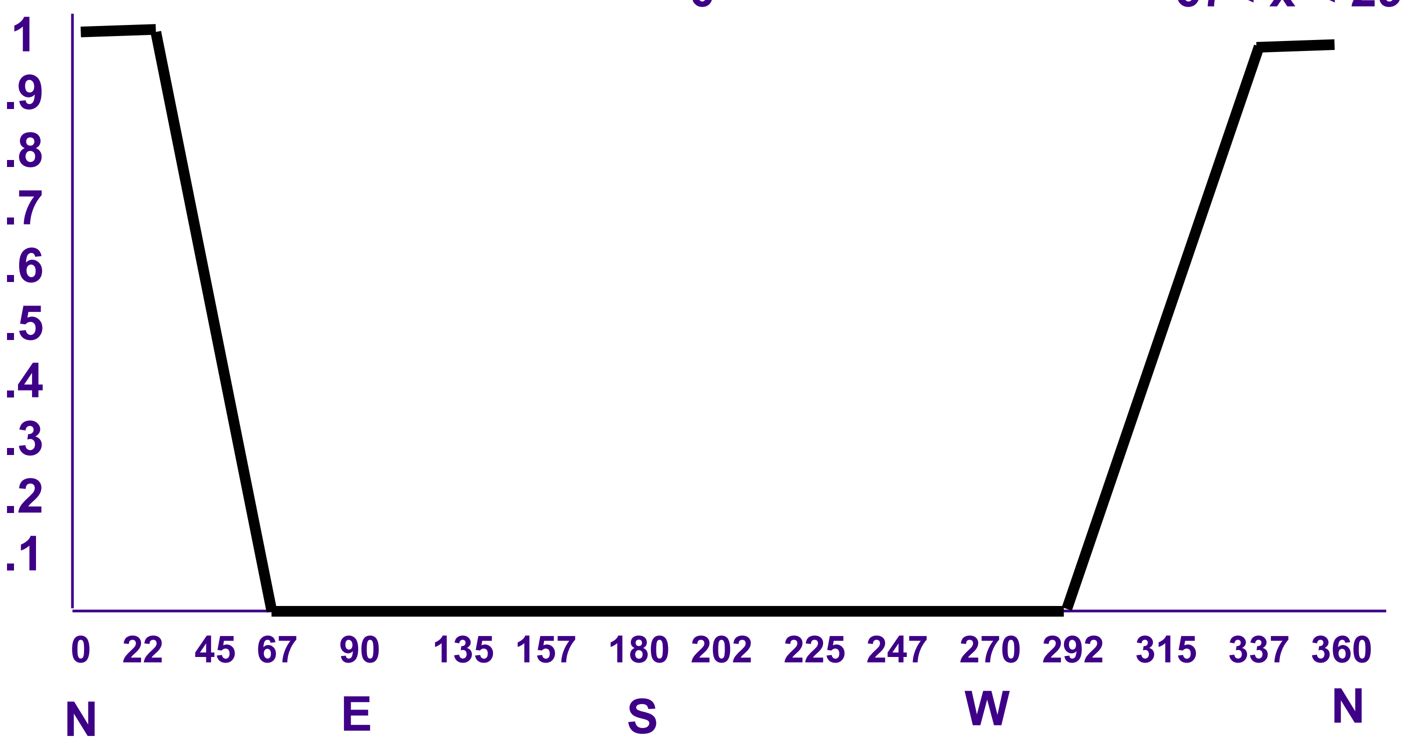
Moderate slope = 0		x < 20
	(x-20) / 80	20 <= x <= 100



Aspect membership function

"Aspect is favorable when the terrain is oriented towards north(0 to 22, 337 to 360), northeast(22 to 67) , northwest (292 to 337)"

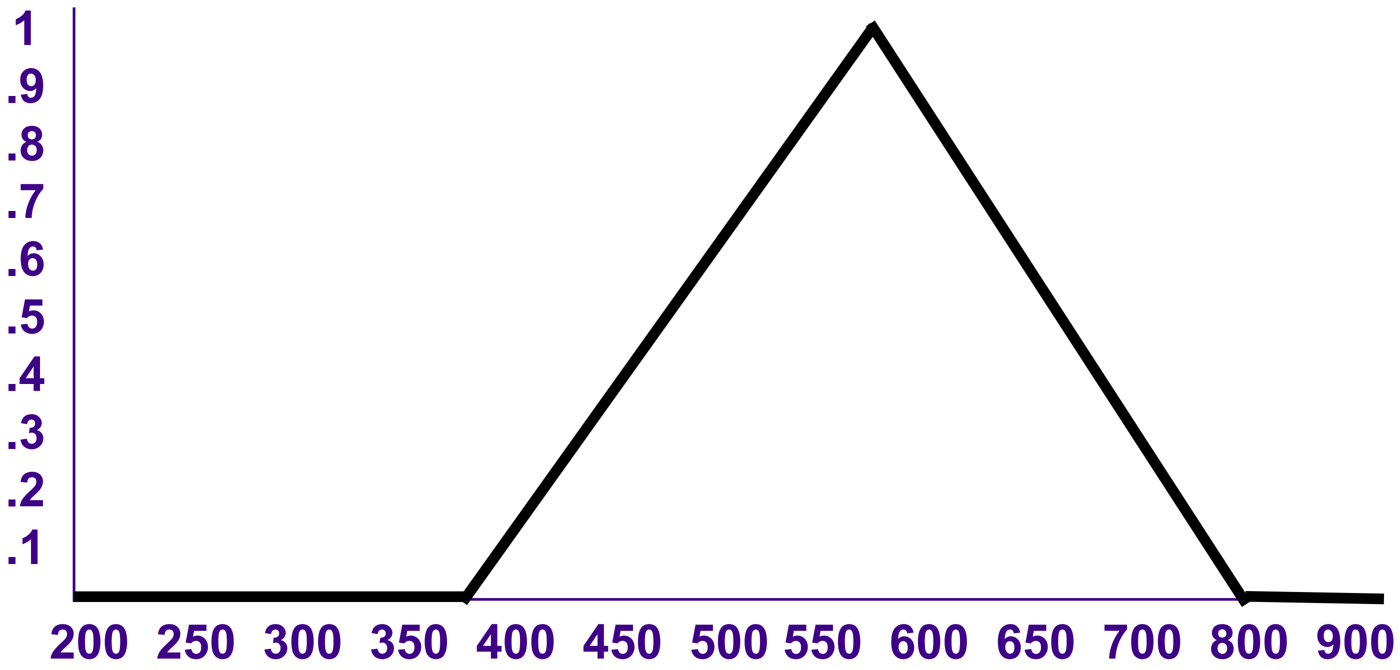
$$
\text{Favorable Aspect} = \begin{cases} 1 & 0 <= x <= 22 \text{ or } 337 <= x <= 360 \\ (x - 22) / 45 & 22 < x <= 67 \\ (x - 292) / 45 & 292 < x < 337 \\ 0 & 67 < x < 292 \end{cases}
$$



Elevation membership function

"preferred elevation is between 380 and 800 meters"

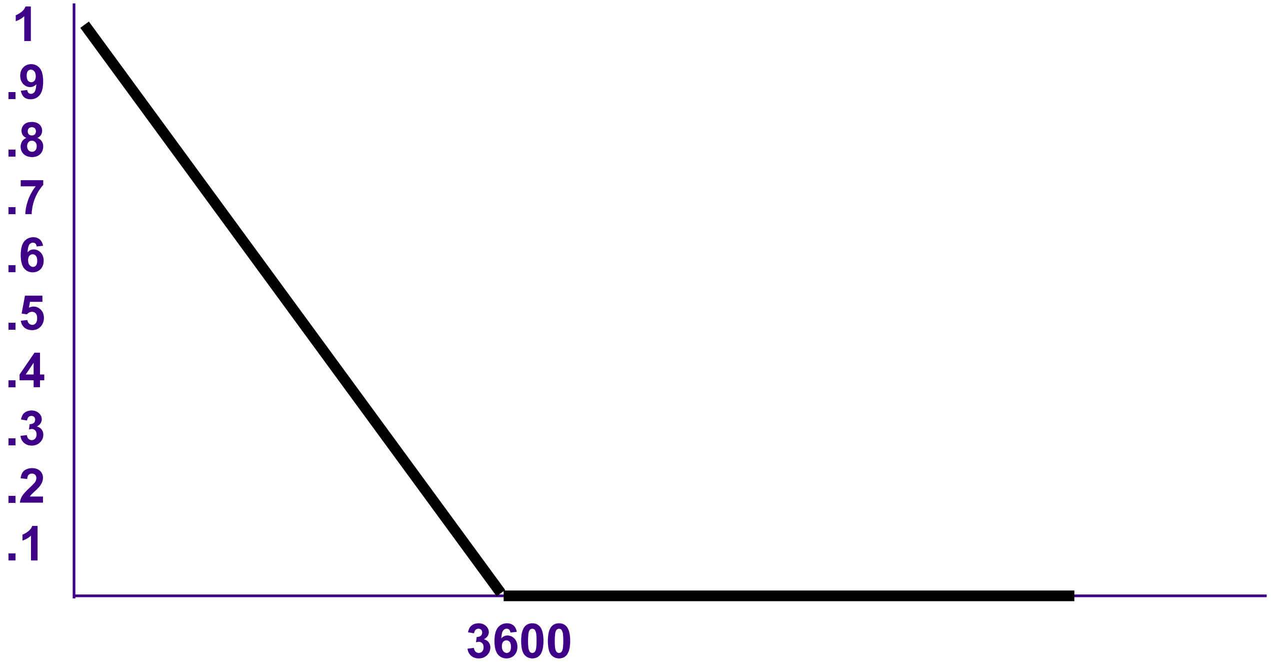
high elevation =		
	0	380 < x or x > 800
	(x – 110) / 210	380 <= x <= 590
	(800 - x) / 210	590 < x <= 800



Water membership function

"It is favorable to be within 3600 meters of water"

Preferred distance =	(3600-x) / 3600	x <= 3600
	0	x > 3600



Lexicographic goal programming (LGP)

- Objective functions are ordered according to their importance
- Given the ordering, the most important function is minimized first, then on the set of optimal solutions with respect to the first function the second function is minimized, and so on, until a unique solution is obtained or all the specified functions are minimized

NOTE: This implies that goals of higher priority must be met before those of lower priority are considered

Preference weight of positive infinity is assigned to a goal of higher priority compared to that of the goal of next-lower priority

Compromise programming (CP)

- Similar to goal programming in that it uses the concept of minimum distance
- A distance based technique that depends on the point of reference or “ideal” point
- Attempts to minimize the “distance” from the ideal solution for a satisficing solution
- The closest one to the ideal across all criteria is the compromise solution or compromise set

CP notes

- The concept of non dominance is used in distance-based techniques to select the best solution or choice of alternative. A solution is said to be non dominated if there exists no other feasible solution that will cause an improvement in a value of the objective or criterion functions without making a value of any other objective function worse (Tecle and Yitayew, 1990).
- The non dominance solution concept, originating with Pareto in 1906, has been one of the cornerstones of traditional economic theory. It is usually stated as the Pareto principle:

> "A state of the world A is preferable to a state of the world B if at least one person is better off in A and nobody is worse off. A state is said to be Pareto optimal or Pareto efficient when there is no other state in which one individual can obtain higher satisfaction without at the same time lowering the satisfaction of at least one other individual" (Just et al., 1982).

CP model

An ideal solution for the compromise programming algorithm, as defined by Tecle and Yitayew (1990), is the vector of objective functions' values, $f^* = (f_1^*, f_2^*, \ldots, f_I^*)$, where the individual maximum values for a criterion i, f_i^*, and minimum or worst value for criterion i, f_i^{**}, are determined using:

(1) $f_i^* = \text{Max}(f_{ij})$, i = 1,2,...I and j = 1,...J

(2) $f_i^{**} = \text{Min}(f_{ij})$, i = 1,2,...I and j = 1,...,J.

CP model

The L_p metric as a compromise solution with respect to p can be expressed as:

$$(3) \qquad \text{Min} \left\{ L_p(A_j) = \left[\sum_{i=1}^{N} (W_i) \left[\frac{(f_i^* - f_{ij})}{(f_i^* - f_i^{**})} \right]^p \right]^{\frac{1}{p}} \right\}$$

- where $L_p(A_j)$ is the distance metric as a function of the decision alternative A_j and the parameter p (Tecle and Yitayew, 1990).
- W_i is the standardized form of the criterion weight, w_i, and represents the decision maker's relative preference structure among the i criteria where the sum of the criteria weights equal one.
- The symbol, f_i^*, represents the ideal or best value for criterion i as described in (1). The notation, f_i^{**}, is the minimum or worst value for criterion i as described in (2).

CP model notes

- In (3), the parameter p can have values from zero to infinity and represents the concern of the decision maker over the maximum deviation (Tecle and Yitayew, 1990) (Duckstein and Opricovic, 1980).
- The larger the value of p, the greater the concern becomes.

  For p = one, all weighted deviations are assumed to compensate each other perfectly.

  For p = two, each weighted deviation is accounted for in direct proportion to its size.

  As p approaches the limit of infinity, the alternative with the largest deviation completely dominates the distance measure (Zeleny, 1982).

Solving the CP model

- To solve the multi criterion problem using the compromise programming algorithm, the vectors of ideal point values f*, and worst values, f**, are determined using (1) and (2).
- These values are then used in (3) to compute the Lp distance values from the ideal points. The preferred alternative has the minimum Lp distance value for each p and weight set that may be used. Thus, the alternative with the lowest value for the Lp metric will be the best compromise solution because it is the nearest solution with respect to the ideal point.
- The parameter p acts as a weight attached to the deviations according to their magnitudes. Similarly *W*i becomes the weights for various deviations signifying the relative importance of each criterion (Romero and Rehman, 1989).

"double weighting scheme"

NOTES on CP

- **Compromise programming resembles goal programming**
- **The solution for goal programming corresponds to the solution for compromise programming when p = 1 if the same weights and aspiration levels are chosen.**
- **Chebyshev goal programming corresponds to the min-max criterion, that is to compromise programming if p = oo**

CP advantages

- Simple conceptual structure
- Simplicity makes it particularly useful for spatial decision problems in which decision makers tend to rely on their intuition and insight
- The set of preferred compromise solutions can be ordered between the extreme criterion outcomes

  (consequently, an implicit trade-off between criteria can be performed)

CP disadvantages

- Except at the two extremes where p = 0 and p = oo there is no clear interpretation of the various values of the parameter p.
- Therefore, the selection of the “best” alternative within the reduced set of compromise alternatives must be made based on intuition

GP advantages

- The major advantage of goal programming is its computation efficiency
- It allows us to stay within an efficient linear programming computational environment
- The weights, aspiration levels, preemptive priorities can be changed during the analysis as the decision maker’s knowledge of the decision problem changes. (Interactive Programming)

GP disadvantages

- GP requires that the decision maker specify fairly detailed a priori information about his or her aspiration levels, preemptive priorities, and the importance of goals in the form of weights
- In many complex problems, it is difficult (or even impossible) for the decision maker to provide the precise information required by these methods

NOTE: studies have shown that decision makers find it relatively easy to specify ordinal rankings for goals, but they are unable to derive meaningful preference weights on a cardinal scale

These difficulties are aggravate further when the goals are unrelated to each other.

GP disadvantages cont’

- The resulting solutions to GP may be dominated (a better solution may exist in terms of some or all of the objectives than the solution obtained through GP)
- There is a tendency to generate inefficient solutions, the GP approach does not attempt to use additional information to find an efficient solution.

NOTE: Having specified an attainable set of aspiration levels, the analyst or decision maker gets exactly what her or she wants even if better decision outcomes are possible

GP models only yield decisions that have the closest outcomes to the aspiration levels specified.

Summary points

- Making decisions is part of everyday life
- Often we learn from what has worked or not worked in the past and use the experience to help us make future decisions.
- If we don’t have prior experience, we often can benefit from a systematic and comprehensive approach to decision making
- The multi-objective methodology with goal programming is such an approach

How to select an optimization model?

Fortunately, Hobbs (1983) outlined the following criteria as a checklist in selecting a technique:

- How accurately does a method represent the decision maker's value structure?
- How many objectives and alternatives can be included?
- Are sensitivity analyses easily made?
- Can the values of several decision makers be considered?
- Does the decision maker comprehend the purpose, assumptions, and wordings of the method?
- In choosing weights or other parameters, do procedures elicit responses that are accurate and stable reflections of the decision maker's values?
- Do decision makers have confidence in the results?

Future for MODM

I feel the use of MODM as a decision making tool will increase in two main areas.

1. The first is as a complement to profit maximization, cost minimization, net benefit, cost effectiveness, and technological-based optimization models which find the solution to a single objective or criteria. The combination of MCDM and single objective optimization models provides a very strong information base in which to make decisions.
2. The second area is as a complement to geographic information systems (GIS) to increase their effectiveness as a complete decision support system. The power of MCDM and GIS together provides an interactive tool to visualize decision alternatives before they are actually made.

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